Evanescent-mode propagation and quantum tunneling

A. Enders and G. Nimtz
II. Physikalisches Institut, University of Cologne, D-5000 Köln 41, Germany
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The tunneling of particles is in direct analogy with the transmission of evanescent electromagnetic waveguide modes as has been shown quite recently. We compare experimental data of an electromagnetic wave packet traversing an evanescent waveguide region with theoretical values derived for particle tunneling through a rectangular potential barrier. The transmission time was deduced by transformation of the experimental frequency data to the time domain. The data are in agreement and reveal superluminal wave-packet velocities for opaque evanescent regions.

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More than half a century has gone by since the introduction of the Schrödinger equation. The equation has given a description for the tunneling probability of wave packets; however, up to now there is no generally acknowledged method available for calculating a tunneling time, in spite of the importance of this quantity for modern microelectronic tunneling devices [1,2].

It has been pointed out that particle tunneling and the propagation of evanescent electromagnetic modes in a waveguide are formally similar; see, e.g., Refs. [3,4]. Previously, Martin and Landauer have emphasized that the propagation of an evanescent electromagnetic wave packet in a waveguide is equal to electron tunneling through a rectangular potential barrier [5]. The matching conditions imposed on the electric and magnetic fields for transverse electric, and for transverse magnetic waveguide modes at the interface between a propagating and an evanescent region are equivalent to those of the particle tunneling problem at a rectangular potential barrier. However, in spite of the formal similarities, one has to keep in mind the striking differences in the interpretation of a probability amplitude of a single quantum-mechanical process and of classical field amplitudes.

A short time after the tunneling experiment across thin insulating layers by Giaever [6] and others, Hartman studied tunneling of wave packets [7]. He calculated the tunneling time for a Gaussian wave packet crossing a rectangular barrier being given by the derivative of the phase delay to incident momentum. This is a so-called phase-time approach, which corresponds to the group velocity [1,7]. His phase-time approximation is based on a linear superposition of incident, reflected, and transmitted wave functions at the barrier boundaries. As shown in Fig. 1, the numerical results can be divided into three regions of barrier transition time depending on barrier thickness. For very thin barriers, the packet's transmission time is longer than the *equal time*, which represents the time for the incident packet to traverse a vacuum distance equal to the barrier thickness. For thicker, i.e., opaque barriers, the transmission time becomes independent of barrier thickness. For very thick barriers (not shown in Fig. 1), the transmitted wave packet is badly distorted, with the greatest contribution coming from Fourier components corresponding to energies just above the top of the barrier, where the transit time is approximately the equal time [7]. The intermediate case includes the possibility of superluminal particle velocity [2,5].

Recently, we have carried out experiments in order to investigate the propagation properties of evanescent electromagnetic modes in rectangular waveguides [8]. The wave number of the basic mode in a rectangular waveguide is given by the dispersion relation [3,4]

$$k^2=(2\pi\nu/c)^2-(2\pi\nu_c/c)^2 , \tag{1}$$

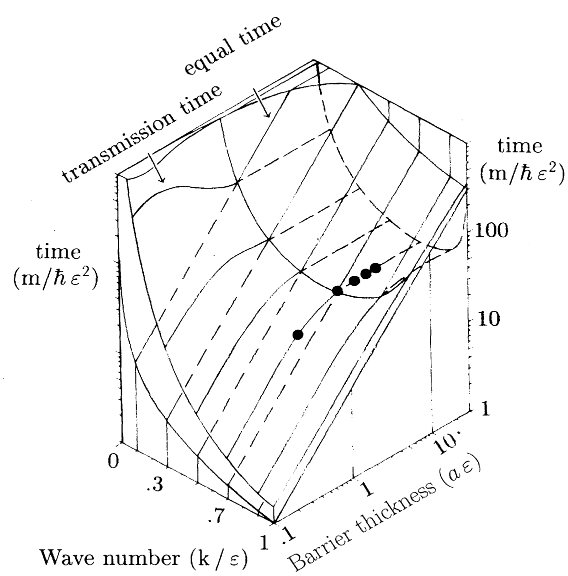

FIG. 1. Graphs of the calculated particle transmission time as a function of barrier thickness [7]. Where m is the particle mass, $\hbar$ the Planck constant, and k/ϵ is the incident wave number normalized to ϵ, the wave number equivalent to the potential barrier height. The dots represent the appropriately scaled experimental data of transmission time of evanescent electromagnetic waves [8]. Experimental parameters are as follows: center frequency of the Gaussian-like wave packet $\nu=8.7$ GHz, $\nu_{c1}=6.56$ GHz, $\nu_{c2}=9.49$ GHz, $a=10$, 40, 60, 80, and 100 mm; for more details see Ref. [8].

where ν is the frequency, c is the velocity of light, $\nu_c = c/2b$ is the cutoff frequency of the waveguide, and b is the waveguide width (see Fig. 2). Thus, for $\nu \leq \nu_c$, the wave number is purely imaginary and this very mode is called an evanescent electromagnetic wave.

The transmission of an evanescent region was measured as a function of frequency, and with the length a of the region as parameter [8]. The geometrical discontinuities at the waveguide transitions have a phase behavior similar to the ideal one-dimensional guide, having only a change in the refractive index, as discussed in Ref. [5]. For instance, at a frequency of 8.7 GHz, the phase shift calculated for the ideal waveguide is $-23°$, whereas in the experiment with the geometrical discontinuities, $-15°$ was measured [8], with similar deviations in the whole frequency interval. This deviation has a minor influence on the evaluated time-domain data. On the other hand, the transmission amplitude was always dominated by the attenuation of the cutoff section. Deviations from the ideal one-dimensional waveguide due to the geometrical discontinuities were not resolvable.

The time crossing the evanescent region was obtained from the frequency-domain-to-time-domain transform according to the relation

$$F'(t) = \int_{\nu_1}^{\nu_2} A(\nu) T(\nu) \exp(i 2\pi \nu t) d\nu \ , \qquad (2)$$

where $A(\nu)$ is the inverse Fourier transform of the initial Gaussian-like reference wave packet and $T(\nu)$ the frequency-dependent transmission function within the frequency range (ν_1, ν_2). The experiment and its analysis are presented in Ref. [8]; here we want to emphasize that in the experiment, nearly *ideal* conditions are established: the stationary transmission and reflection coefficients at each frequency point are determined as if the source and the detector do not interfere with the evanescent region. This is realized by the process of calibration. From the electrotechnical point of view, this is a procedure to determine all systematic errors in the whole setup. This special procedure allows the evaluation of the scattering parameters of the cutoff waveguide section from measured data with a very good accuracy (better than $\pm 1°$ in phase and ± 0.2 dB in amplitude). From the physical point of view, this makes the source and the detector ideal and suppresses their interference with the cutoff waveguide; i.e., a perfectly matched device is equivalent to an infinitely long transmission line. If these error-corrected scattering data are transformed to the time domain via Eq. (2), the launching and detection of the corresponding wave packets happens without interference, too; i.e., the source and detector behave as if they have an infinite distance from the cutoff section.

The traversal times obtained by this transform—a procedure that works correctly for propagating waves with spatial oscillations in order to determine the time a signal travels along a transmission line or is reflected at a discontinuity of a circuit—equals Hartman's theoretical data for particle tunneling as displayed in Fig. 1. For a short evanescent region $a\epsilon \leq 1$, the crossing time is longer than the equal time, and ϵ is the wave number equivalent

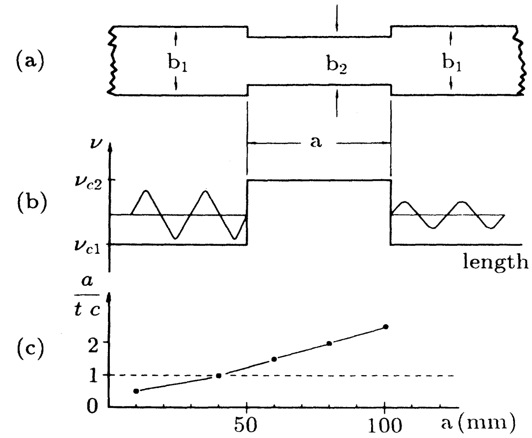

FIG. 2. (a) Top view of the rectangular waveguide structure with widths $b_1 = 22.96$ mm and $b_2 = 15.80$ mm, corresponding to cutoff frequencies $\nu_{c1} = 6.56$ GHz and $\nu = 9.49$ GHz; the studied basic TE_{01} mode does not depend on the waveguide height [3,4]. (b) Graph of photon energy, i.e., frequency vs waveguide length. The investigated frequency range has been $\nu_{c1} \ll \nu \ll \nu_{c2}$; accordingly, the narrow waveguide is operated in the evanescent mode regime. (c) Normalized transmission velocity a/tc vs evanescent region thickness.

to the barrier height of the evanescent region $[\epsilon = \pi(1/b_2 - 1/b_1)]$. For $a\epsilon \geq 1$ (opaque barrier), the crossing time becomes shorter than the equal time; the data are presented as dots in Fig. 1 for photon wave number corresponding to 0.7ϵ. The signal velocity for crossing the evanescent region obtained from the relation $v = a/t$, with t the crossing time and a the region length, are shown in Fig. 2. In the same figure, experimental details of the evanescent-mode experiment are also given.

Hartman's calculated phase time, as well as our experimental data, are essentially asymptotic in character, since they are derived as asymptotic characteristics for completed scattering events involving wave packets narrow in k space [1]. (In our study, the frequency width of 0.5 GHz used corresponds to a packet width in x space of 0.1 m, a value being shorter than the infinite source and detector distances to the evanescent waveguide.)

There are two remarkable results: (i) the frequency-domain-to-time-domain transform for electromagnetic modes yields traversal times that agree with Hartman's wave-mechanical calculation for a particle, and (ii) with increasing length a of the evanescent region, the group velocity extrapolated for this region exceeds the velocity of light. The traveling through an evanescent region appears to be done in zero time, a problem that was recently studied for particle tunneling by Low and Mende [9].

Summing up, evanescent electromagnetic waves propagate superluminally in opaque regions since the traversal time is independent of the region's length [8]. The electromagnetic-mode experiment is assumed to correspond directly to particle tunneling. Obviously, Hartman's model describes the tunneling of both particle and electromagnetic wave packets.

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